Diversification in Funds of Hedge Funds: Is It Possible to Overdiversify?[†]

Stephen J. Brown[*], Greg N. Gregoriou,[**] and Razvan Pascalau[***]

Abstract

Many institutions are attracted to diversified portfolios of hedge funds, referred to as Funds of Hedge Funds (FoHFs). In this paper we examine a new database that separates out for the first time the effects of diversification (the number of underlying hedge funds) from scale (the magnitude of assets under management). We find with others that the variance-reducing effects of diversification diminish once FoHFs hold more than 20 underlying hedge funds. This excess diversification actually *increases* their left-tail risk exposure once we account for return smoothing. Furthermore, the average FoHF in our sample is more exposed to left-tail risk than are naïve 1/*N* randomly chosen portfolios. This increase in tail risk is accompanied by lower returns, which we attribute to the cost of necessary due diligence that increases with the number of hedge funds. (*JEL* G11, G29)

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[*] Stephen J. Brown is the David S. Loeb Professor of Finance at New York University Stern School of Business, 44 West 4th Street, Suite 9-190, New York, NY 10310, Email:sbrown@stern.nyu.edu, Tel: 212-998 0306, Fax: 718-981 7239. He is also a Professorial Fellow at the University of Melbourne.
[**] Greg N. Gregoriou is Professor of Finance at State University of New York (Plattsburgh) 101 Broad Street, Plattsburgh, New York 12901. E-mail: gregorg@plattsburgh.edu; Tel: 518-564-4202; Fax: 518-564-4215. He is also an EDHEC Business School Research Associate in Nice, France.
[***] Razvan Pascalau is Assistant Professor of Economics and Finance at State University of New York (Plattsburgh) 101 Broad Street, Plattsburgh, New York 12901. E-mail: rpasc001@plattsburgh.edu; Tel: 518-564-4193; Fax: 518-564-4215.

Over the past ten years, nearly every financial institution, endowment fund, and pension fund has increased their exposure to alternative investments through Funds of Hedge Funds (FoHFs).[1] This has led to a dramatic growth in the number of FoHFs and their relative importance in the hedge fund industry. Before the recent financial crisis, many institutions were envious of the success of the Yale and Harvard endowments investing in alternative investments, where the returns appeared to exceed the average performance of endowment funds, traditional stock and bond investment portfolios, as well as major market indexes. However, there is very little information available about hedge funds. Indeed, the 1940 Investment Company Act, which allows hedge funds an exemption from Securities and Exchange Commission (SEC) registration and oversight, includes a ban on general solicitation. Many hedge funds interpret this ban as a mandate that they not disclose any information about their activities, viewing such disclosure as a form of solicitation. For this reason, many institutional investors concerned about their fiduciary responsibilities turned to FoHFs. These FoHFs charge an additional layer of fees to assemble a pre-packaged and diversified portfolio of hedge funds and do all necessary due diligence on behalf of their customers.

Many FoHFs accentuate the importance of diversification. Indeed, Samuelson (1967) argues that diversification should be as widely spread as possible. The fact that hedge funds generate returns that have a low correlation with returns on other more traditional asset classes is presented as a major argument in favor of hedge fund investment. Indeed, many FoHFs diversify extensively, with many hedge funds represented in their portfolios. However, this argument does not bear up upon closer scrutiny. Many recent studies have shown that an institutional investor with $100 million to invest can obtain all the diversification required by simply investing in only 10 to 20 hedge funds. Our results strongly support this conclusion.

The reason institutional investors turn to FoHFs is that the necessary due diligence associated with hedge fund investments is both difficult and expensive. Under the Prudent Investor laws of most states,[2] fiduciaries may delegate due diligence responsibilities to responsible third parties, and for this reason are willing to pay 1.5% fees to FoHFs to do something they could not do themselves without involving these intermediaries. This is highlighted in Brown, Fraser, and Liang (2008), who argue that the difference in returns between FoHFs with limited assets under management and larger FoHFs represent economies of scale almost entirely attributed to the fixed cost associated with necessary operational due diligence, a finding further supported in Brown et al. (2008, 2009). Additionally, each due diligence report comes at a hefty price tag.[3] These earlier studies did not have access to information on the number of underlying funds, and obviously the necessary due diligence costs rise with the number of underlying funds in a FoHF as they decrease as a percentage of assets under management. The evidence is strong that there are increasing returns to information. Given that hedge fund information is scarce and costly, the best hedge funds to acquire information on are the ones the FoHFs expect to hold. This information, derived from private information sources, will determine which funds the FoHFs hold. Van Nieuwerburgh and Veldkamp (2010) show that the investor, in this case the FoHFs, will specialize in their investment portfolios because they prefer to hold assets they are informed about. This implies that the FoHFs should be relatively undiversified.

Diversification is of course not simply a matter of holding many funds. Given the well-understood arguments by Lo (2001) and Lim et al. (2006) that hedge funds are providers of liquidity who earn rents on the provision of this liquidity, investors should not expect them to do well in liquidity crises. This may explain Agarwal and Naik's (2000) finding that hedge fund strategies on average have large positive correlations during bear market conditions. Boyson, Stulz, and Stahel (2010) argue that exposure to liquidity shocks is a major factor responsible for hedge fund contagion. FoHFs that efficiently diversify away business risk considerations do not necessarily provide any protection against the common factor represented by left tail market risk exposure. Indeed, FoHFs diversification concentrates this risk. We would expect that the larger the number of underlying hedge funds in the FoHFs, the more exposed the FoHFs should be to these negative market conditions. Indeed, using data prior to the financial crisis of 2008-2009, Brown and Spitzer (2006) find that FoHFs are more exposed to tail risk exposure than are other hedge fund strategies. It is not surprising that according to the most recent Lipper TASS database that 22% of FoHFs and 30% of all multi-strategy funds failed during the recent credit crunch crisis, a fraction as large as or larger than the fraction of failures in any other hedge fund category. This instance of “we all fall down together" suggests a further and careful examination of the risk characteristics of FoHFs during the recent period of financial crisis is warranted.

Our study is the first to examine the actual experience of diversification within a large cross section of FoHFs. Using a unique information source that provides data on the number of underlying hedge funds we are able to show the effects of diversification on both risk and potential performance of FoHFs. Over 34% of our sample controlling 60% of the U.S. Dollar assets under management hold more than 25 underlying funds. In some cases the diversification is very extensive, with 89 U.S. dollar funds controlling 14% of the total FoHFs assets under management, and in the same time holding more than 50 underlying hedge funds each. It is hard to understand why so many FoHFs are as well diversified as they appear to be. Using this data, we are able for the first time to examine the adverse consequences that follow from the excess diversification that appears to be a characteristic of the FoHFs industry.

The paper is organized as follows. Section 1 reviews the existing literature on FoHFs diversification. Section 2 examines the available data on FoHFs for the period 2000-2010 and analyses the role that the number of underlying hedge fund managers has on the resulting measures of risk and return. Section 3 concludes.

1. Literature Review

During the last 12 years numerous papers have addressed the issue of how many underlying hedge funds constitute sufficient diversification for FoHFs. These were mainly hypothetical exercises involving naïve 1/*N* strategies using randomly selected hedge funds chosen from one of the major hedge fund databases. However, these studies do not examine the actual diversification strategies of FoHFs since comprehensive data on the number of underlying hedge funds has only recently become available.

One of the first papers to tackle this issue was Henker (1998). By means of a naïve diversification approach, he selected a small set of hedge funds at random during the 1992 to 1997 bull market. He concludes that a minimum of at least ten funds is necessary to achieve meaningful diversification in a FoHFs. Subsequently, using an argument derived from Elton and Gruber (1977), Park and Staum (1998) demonstrate that at least 20 hedge funds are required in a FoHFs to eliminate 95% of the diversifiable risk in the portfolio. In addition, to shed further insight Peskin et al. (2000) construct 1,000 randomly selected portfolios of hedge funds to determine whether the characteristics of hedge fund indexes can be duplicated over a longer period (1990-2000). During the portfolio construction process, they raise questions such as whether to include hedge funds with considerable track records and small or large sized funds. They conclude that a portfolio of 20 hedge funds will likely produce similar characteristics to those of hedge fund indexes.

Using a final sample of 455 hedge funds during the June 1994 to May 2001 period, Amin and Kat (2002) generate 500 equally-weighted portfolios randomly while addressing the biases in the dataset. At the outset they believe that smaller portfolios of 1-20 hedge funds are superior to a large number of funds. Their results, in line with Lhabitant and Learned (2002, 2004), show that diversification reduces standard deviation. In addition, they provide some evidence that skewness decreases through diversification to a level consistent with that of the skewness of the S&P 500 index and increases portfolio correlations with the equities markets more generally. Kurtosis remains relatively unchanged. The results of their simulation suggest that approximately 15 funds are required to attain optimal diversification and to closely resemble the hedge fund universe. Using similar procedures, Lhabitant and Learned (2002, 2004) examine the properties of naïve 1/*N* hedge fund strategies using data from 1990 to 2001 and four sub-periods. They conclude that 5-10 managers offers the greatest diversification benefits in single strategy FoHFs and 10-15 managers for multi-strategy FoHFs.

Brown, Goetzmann, and Liang (2004) and Brown, Fraser, and Liang (2008) offer two different reasons why increasing diversification may lead to poor performance. First, Brown, Goetzmann, and Liang (2004) argue that since the FoHFs receives returns net of management and incentive fees, if one or several funds do well they will incur incentive fees regardless of how well the FoHFs as a whole does. In fact, the higher the number of funds in a FoHFs, the larger the accumulation of incentive fees at the fund level that are passed through the FoHFs vehicle. These fund-level incentive fees then become a fixed charge payable by the investor (whether or not the FoHFs does well or poorly). Second, Brown, Fraser, and Liang (2008) argue that since a standard operational due diligence provided by private due diligence companies costs a considerable amount on a per fund basis, the larger the number of funds, the larger the cost to the FoHFs. While due diligence charges should be covered by the management fee, there is an incentive for small funds to skip due diligence particularly when its cost exceeds the management fees the FoHFs can charge.

Due to the well-documented hedge fund deaths over the last few years, many investors have sought shelter in funds with many underlying hedge funds for safety; however, this may be a counterintuitive approach. Large FoHFs appear to be embraced by investors more than their smaller counterparts due to their extensive track records and the "safety issue." However, Füss, Kaiser, and Strittmatter (2009) argue that larger FoHFs have lower standard deviation and inferior returns. In essence, selecting a FoHF with many managers may offer safety but at a cost of inferior performance. There is only very limited evidence of hedge fund performance persistence (Brown, Goetzmann, and Ibbotson 1999; Agarwal and Naik 2000) and this might suggest that an excessive number of hedge fund managers in a FoHFs may be counter-productive — there are not that many first-rate hedge fund managers.

2. Data Analysis

We use monthly net returns of all fees supplied by the Barclay Hedge database from January 2000 until March 2010[4] excluding the first 18 months of data since fund inception to address well-known backfill bias issues with these fund databases.[5] The Barclay hedge fund database is the only database that reports the number of underlying hedge fund managers in FoHFs. On the other hand, it does not report dead funds prior to January 2000 and for this reason we are limited to data subsequent to that date to address possible survivorship bias issues.

2.1. Descriptive summary

We have information on 3,767[6] FoHFs, out of which 58 are closed to new investors, 1,324 are open, 933 have been delisted, 915 have been liquidated, and 477 which stopped reporting by March 2010 for reasons not reported to the database. Table 1 reports descriptive statistics for those U.S. dollar funds in the sample that report at least 12 months of performance for the period January 2000 through March 2010.[7] The evidence seems to suggest that performance measured in terms of after fee net return peaks at between 7-10 underlying hedge funds. Many of these funds are quite small, with a median size less than about $20M. Between a third and a half of all FoHFs are very well diversified, with more than 20 underlying hedge funds. These funds tend to be significantly larger, with more than half having more than $100M assets under management.

[Insert Table 1 approximately here]

A number of prior studies have examined the diversification benefits that can arise through a diversified hedge fund strategy. Elton and Gruber (1977) provide a simple formula that shows how the standard deviation of naïve $1/N$ portfolio strategies varies with N, the number of securities in the portfolio. However, Getmansky, Lo, and Makarov (2004) argue persuasively that hedge fund return standard deviations are biased down because of smoothing and illiquidity. No one to our knowledge has examined the role of smoothing on the apparent diversification benefits of FoHFs. Because this issue is difficult to address within the Elton and Gruber (1977) framework, we consider an older technology due to Evans and Archer (1968).

The Barclay Hedge database contains 5,639 U.S. dollar hedge funds (excluding FoHFs) alive and dead with at least 24 months of holding period returns net of fees. We construct 60 months of data for a hypothetical FoHFs by first randomly selecting with replacement a fund from this database. The last valid return defines the end of the 60-month period. We then replace this fund with other randomly selected funds if the fund has less than 60 months of data or the holding period return is otherwise unavailable. In this way we avoid the survivorship bias referred to by Amin and Kat (2002) in this context. We then choose at random another fund with returns within this 60-month period replacing it as necessary to build up a second contemporaneous time series of returns. We repeat this exercise to construct equally-weighted hypothetical portfolios of 2 to 350 underlying hedge funds. In this way we can compute both the standard deviation for each hypothetical fund and the standard deviation appropriately accounting for smoothing. We first measure variance assuming the smoothing occurs on a quarterly basis and then compute the standard deviation of returns expressed on an equivalent monthly basis.[8] We then repeat this experiment 25,000 times. The average value of standard deviation both with and without smoothing correction is reported in Figure 1, where it is seen that failure to correct for smoothing leads to a bias of about 60 to 80 basis points of standard deviation. However, regardless of whether we correct for smoothing or not, a well-diversified hedge fund strategy has less than half the risk of the S&P 500 index measured over the same period of time.

Superimposed on Figure 1 we also provide the median value of the standard deviation measured for the set of U.S. dollar hedge funds included in our sample, as a function of the number of underlying hedge funds. We see that the average FoHFs does better in terms of risk diversification than the naïve $1/N$ strategy for all but the most diversified FoHFs strategies.[9] Yet in either case, the gains from diversification tend to peter out at between 10 and 20 underlying hedge funds.[10]

[Insert Figure 1 approximately here]

To the best of our knowledge only one study has examined the effect of diversification on hedge fund tail risk exposure[11] and there are no studies that examine the importance of properly accounting for smoothing in measuring this tail risk. The same technique used to construct Figure 1 can be employed to examine the effect of diversification on skewness and kurtosis. FoHFs that efficiently diversify away business risk considerations do not necessarily provide any protection against the fact that many hedge funds are exposed to left tail market risk exposure, which Lo (2001) and Lim et al. (2006) attribute to the fact that many hedge funds earn rents from the provision of liquidity and thus lose heavily in liquidity crises. Indeed, FoHF diversification may concentrate this risk. This finding is supported in Figure 2, which shows that the magnitude of negative skewness is an *increasing* function of diversification, whether or not we account for smoothing.[12] The asymptote as the number of underlying funds grows large is within the range of the skewness of various measures of market liquidity that have appeared in the recent literature.[13] This result suggests that tail risk exposure is non diversifiable, at least when the investor is limited to hedge fund strategies that are all exposed to the common factor of liquidity exposure.[14]

Not only does tail risk increase with the number of underlying funds in the portfolio, the average FoHF is more tail risk exposed than is one constructed from a random choice of funds. Figure 2 gives the median values of skewness within each range of the number of underlying funds, and in each case except for the most diversified FoHFs, skewness is more negative than skewness of those constructed from a random choice of funds. The fact that FoHFs are more subject to left-tail risk than in a random 1/*N* portfolio strategy is consistent with the view that both hedge funds and FoHFs are attracted to strategies that increase short-term performance measures at the expense of increasing tail risk exposure.[15] We see a similar pattern involving kurtosis. What is interesting here is that return smoothing obscures the fact that diversification increases tail risk, at least as measured by kurtosis.[16]

[Insert Figure 2 approximately here]

[Insert Figure 3 approximately here]

We consider a simple and robust measure of tail risk neutrality considered by Brown and Spitzer (2006) to examine further the extent to which a common exposure to market tail risk explains the extent to which tail risk is an increasing function of diversification. By considering the frequency with which hedge fund returns in their lowest decile correspond with market returns in their lowest decile, they find that FoHFs are more exposed to market tail risk than is the average hedge fund. They attribute this finding to the fact that while diversification eliminates idiosyncratic business risk, it concentrates exposure to left tail market risk events, an instance of "we all fall down together."

Figures 4, 5, and 6 replicate Figures 1, 2, and 3 where we simply exclude from consideration all months for which the return on the S&P 500 index in excess of the one-month T-bill is less than the tenth percentile of the distribution of this quantity, a well-defined market left tail event. If hedge fund returns were in fact left tail neutral, excluding such months should have no effect on the distribution of hedge fund returns. Yet we see a considerable decline in the magnitude of standard deviation, skewness, and kurtosis. Furthermore, the magnitude of skewness and kurtosis are now a *decreasing* function of the degree of diversification. This result confirms the conjecture that increases in left tail exposure with diversification is a result of exposure to left tail market risk exposure. Indeed, once we account for smoothing,[17] absent market left tail events, well-diversified portfolios of hedge funds have skewness and kurtosis similar to that of Gaussian distributed returns. Again, absent such market risk events, the FoHFs in our sample have lower standard deviation and kurtosis than do randomly chosen 1/*N* portfolios of hedge funds. On the other hand, these FoHFs have low and mostly negative skewness even when we exclude from consideration market left tail events. This provides further evidence that FoHFs actively seek out hedge funds with left-tail risk exposure.

[Insert Figure 4 approximately here]

[Insert Figure 5 approximately here]

[Insert Figure 6 approximately here]

2.2. Analysis of performance

The variance-reducing attributes of diversification tend to peter out at about 10 to 20 funds. Almost half of the FoHFs in our sample are more diversified than this. FoHFs may believe that excess diversification reduces tail risk exposure, but the data suggests that tail risk *increases* with the degree of diversification. Perhaps overdiversification can be explained by economies of scale that lead to improvements in performance as the fund becomes more diversified, although this does not appear to be the case.

We compute performance for each hedge fund with at least 36 months of performance history relative to the Sharpe ratio, the seven factor Fung and Hsieh (2004) benchmarks as the alpha in the regression:

$$r_{it} = \alpha_i + \beta_{1i}PTFSBD + \beta_{2i}PTFSFX + \beta_{3i}PTFSCOM + \beta_{4i}(Equity\ Mkt\ Factor) +$$

$$\beta_{5i}(Bond\ Factor) + \beta_{6i}(Credit\ Spread) + \beta_{7i}(Size\ Spread) + \epsilon_{it}, \qquad (1)$$

where i=1, …, N funds, t=1,…,T months, r_{it} is the fund return in excess of the one-month T-bill return, $PTFSBD$ is the return of the PTFS bond lookback straddle, $PTFSFX$ is the return of the PTFS currency lookback straddle, $PTFSCOM$ is the return of the commodity lookback straddle,[18] $Bond\ Factor$ is the total return for 10-year government bonds reported in CRSP in excess of one-month T-bill return, $Equity\ Mkt\ Factor$ is the S&P 500 index monthly total return in excess of the one-month T-bill rate, $Credit\ Spread$ is the difference in total return between the total return on the Bank of America Merrill Lynch U.S. Corporate Bond Index and the total return on the 10-year government bonds reported in CRSP,[19] and $Size\ Spread$ is the CRSP small decile return less large decile return. Given the extensive use of leverage in many hedge fund strategies, Agarwal and Naik (2000) argue instead for the use of the appraisal ratio (the ratio of alpha to the standard deviation of the residuals in this regression), which is leverage invariant.[20] We then examine the relationship between these different measures of performance and the degree of diversification by regressing the estimate of alpha against dummy variables indicating the degree of diversification, the log of assets under management, and the maximum funds available for due diligence, given as the assets under management per underlying fund multiplied by the management fee.

Getmansky, Lo, and Makarov. (2004) present convincing evidence of return smoothing and illiquidity sufficient to cause hedge fund returns to be positively autocorrelated. Many hedge funds report returns to databases on a quarterly basis. Indeed, we find strong evidence of an MA(2) process in individual FoHFs returns consistent with smoothing on a quarterly basis either at the constituent fund basis or the FoHFs basis. Asness, Krail, and Liew (2001) argue that one should address this issue by including lagged factors in the alpha regression, and Lo (2008) suggests that this regression be estimated using GLS, assuming that the errors follow a moving average process of order equal to the number of lagged factors in this regression. Table 2 reports results for the Sharpe ratio, adjusting for quarterly smoothing and then for the alpha regressions where alpha is estimated on the basis of the Fung and Hsieh factors and two lags of these factors using a full information maximum likelihood procedure assuming the errors follow a MA(2) process. While there is substantial evidence of smoothing in the data, we find that the results do not differ either qualitatively or quantitatively from the results obtained without taking smoothing into consideration.

[Insert Table 2 approximately here]

Table 2 also shows that there is no obvious performance benefit to overdiversification. Regardless of the measure of performance, the highest performing funds have between 17 and 26 funds. In a regression where we include dummies for 2-10 funds, 11-25 funds, and 26-409 funds, we find that diversification beyond 25 underlying funds leads in most cases to a significant reduction in relative performance. Figure 7 illustrates that overdiversification cannot be justified by reference to any potential performance benefits from such a strategy.

[Insert Figure 7 approximately here]

Brown, Fraser, and Liang (2008) find that there are diseconomies of scale among hedge funds, consistent with the assumptions underlying the Berk and Green (2004) model and the empirical results reported in Fung et al. (2008). By contrast however, they find significant economies of scale among FoHFs, which they attribute to the fixed costs of operational due diligence. These results are strongly supported in Table 2. Performance among FoHFs is a strongly increasing function of assets under management. Indeed, this effect dominates the diversification effect. However, there remains a substantial reduction in performance among overdiversified funds when we consider the difference in performance between funds with between 11 and 26 underlying funds and FoHFs that are more diversified than this. We examine the Brown, Fraser, and Liang (2008) hypothesis by including a proxy for operational due diligence given as the maximum available funds for this purpose.[21] We find that that this due diligence proxy is significantly associated with performance. However, even including this due diligence proxy, assets under management are still significantly associated with positive performance.

2.2. Hedge funds, due diligence and death

The results reported so far suggest that is there no apparent benefit in either diversification or return to overdiversification. If anything, overdiversification hurts FoHFs after controlling for assets under management both in terms of tail risk exposure and in terms of performance. Given limited transparency, hedge fund diversification is costly. The performance figures reported in Table 2 are net of fees. Due diligence is a costly enterprise. Brown et al. (2012) report that the minimal fee for due diligence can be as much as $12,500 and then only if the client is willing to share the results of the due diligence with other clients. If the client is not willing to share the results with others, the cost quickly escalates to $50,000.[22] If the FoHF is well diversified, an annual due diligence on each and every fund could quickly consume all of the management fees it charges. This suggests a very simple necessary condition to establish whether a FoHF is performing appropriate due diligence on an annual basis:

$$\frac{U.S.\,Dollar\ Assets\ under\ management}{Number\ of\ underlying\ hedge\ funds} \times Management\ fee > \$12{,}500 \qquad (2)$$

If a FoHFs passes this test, there is of course no guarantee that they are performing necessary due diligence on an annual basis. But if a FoHFs fails this simple test, then it cannot afford to perform an annual due diligence since the total sum of the annual management fees divided up among the constituent funds would not suffice to pay for necessary due diligence. Thus, for each classification we compute the proportion of FoHFs that survive relationship (2) (i.e., the due diligence test). Table 3 illustrates the surprising result that only about a quarter of all FoHFs passes this test. The remainder simply cannot afford to do a due diligence on each of the funds they invest in, at least on an annual and up-to-date fashion. Under the null hypothesis, all funds in the sample pass the simple due diligence test. The large *t*-values reported in Table 4 strongly reject the null hypothesis and suggest that a significant proportion of the funds fail the test.

[Insert Table 3 approximately here]

Brown et al. (2008) argue that due diligence is a source of alpha in a well-diversified hedge fund strategy to the extent that hedge fund failure is predictable and can be avoided by appropriate due diligence. The fact that overdiversified FoHFs cannot afford to do the necessary due diligence in a timely fashion may explain why they have significantly lower performance than other FoHFs.

In extreme cases, the failure to perform necessary operational due diligence can lead to the death of the fund. Table 4 reports the results of an analysis of a Cox Proportional Hazards analysis to examine the causes of the death of funds. Considered by itself, the smaller the fund, as measured by the assets under management, the greater the probability of failure. However, assets under management may simply be a proxy for the ability of the FoHFs to perform necessary due diligence.

Once we control for financial risk (measured by standard deviation and left tail risk) and the affordability of due diligence, we find that the larger the fund, the more likely it is to fail. Controlling for these factors, diversification as expected improves the survivability of the fund. But it is important to note that simply increasing the scale of the FoHFs through diversification without proper regard for due diligence and tail risk exposure is no guarantee that the fund will survive. For this reason, the result is an important caveat to the earlier findings of Brown, Fraser, and Liang (2008), who suggest that fund size (assets under management) alone might give some assurance of survivability.

[Insert Table 4 approximately here]

Of course, funds fail for many reasons. Changes of personnel and a falling off of investor confidence leading to untimely withdrawals are leading causes of failure. Many FoHFs that were in fact feeder funds for Bernard Madoff certainly did not fail as a result of overdiversification. On the other hand, there is no evidence in this data that FoHFs that could afford necessary due diligence actually performed it. Nevertheless, it is striking that there is a significant association between failure and the ability of funds to pay for necessary due diligence.

3. Conclusion

Given the lack of transparency and high cost of information we would expect significant limits to diversification among FoHFs, particularly as the risk reduction benefits tend to peter out between 10 to 20 underlying hedge funds. It is then a puzzle to explain why it is that a large fraction of all FoHFs have more than 25 underlying funds, particularly given that tail risk exposure increases with diversification. In addition, timely due diligence is costly and the cost increases with the number of funds under management. The data show that only a quarter of all well-diversified FoHFs can afford to do annual due diligence on all of the funds they manage, even if the total value of all management fees paid to the fund were expended for this purpose. Not surprisingly, the resulting performance of well-diversified funds lags that of FoHFs with between 10 and 25 funds under management. Furthermore the data indicate that the inability to afford timely due diligence is significantly associated with the failure of the FoHFs.

Overdiversification does not imply meaningful risk reduction, leads to diminished returns and in extreme cases, death of the fund particularly when it becomes too expensive to perform necessary due diligence. If we accept the argument that information costs lead investors to underdiversify, then we cannot explain why it is that FoHFs overdiversify. This is then an interesting institutional anomaly and a subject for further research.

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Table 1: Descriptive statistics - Fund of Funds Statistics by Number of Underlying Funds, Jan. 2000 – Mar. 2010

U.S. dollar Funds	**Average Monthly Fund Returns**					**Assets under Management**		
Number of Underlying Funds	*N*	*Cumulative %*	*Average*	*t-value*	*Median*	*N*	*Average ($M)*	*Median ($M)*
1 – 2	53	3.85%	0.17%	1.49	0.36%	44	$66.31	$20.50
3 – 4	35	6.40%	0.23%	1.61	0.17%	18	$113.71	$15.90
5 – 6	108	14.25%	0.09%	0.96	0.26%	102	$257.47	$26.36
7 – 8	47	17.67%	0.43%	5.01	0.34%	42	$217.02	$10.63
9 – 10	75	23.13%	0.26%	4.74	0.37%	71	$206.44	$19.14
11 – 12	74	28.51%	0.12%	1.39	0.28%	68	$46.43	$26.60
13 – 14	59	32.80%	0.05%	0.40	0.33%	52	$131.61	$55.52
15 – 16	105	40.44%	0.13%	2.17	0.31%	95	$77.90	$25.46
17 – 18	72	45.67%	0.37%	5.49	0.41%	65	$131.47	$47.00
19 – 20	102	53.09%	0.21%	4.95	0.30%	96	$152.41	$39.64
21 – 25	173	65.67%	0.27%	6.50	0.37%	161	$195.96	$71.03
26 – 30	162	77.45%	0.23%	5.41	0.30%	158	$325.19	$130.29
31 – 40	143	87.85%	0.23%	3.16	0.39%	124	$419.84	$129.19
41 – 50	78	93.53%	0.12%	1.57	0.31%	74	$560.38	$108.20
51 – 409	89	100.00%	0.19%	2.98	0.34%	75	$571.71	$171.00
Total	1375							

This table provides descriptive statistics for all U.S. dollar FoHFs in the Barclay Hedge database reporting at least 12 months of returns net of fees.

Table 2: Regressing performance measures on degrees of diversification and on assets under management accounting for quarterly smoothing

Panel 1 **Sharpe_Ratio**

Number of Underlying Funds	*Model 1*			*Model2*			*Model 3*		
	Coefficient	*t-value*		*Coefficient*	*t-value*		*Coefficient*	*t-value*	
2	0.0878	3.23	**	-0.5075	-8.10	**	-0.5112	-7.64	**
3-4	0.1725	2.19	*	-0.4555	-9.10	**	-0.4250	-8.65	**
5-6	0.1011	2.62	**	-0.4761	-6.53	**	-0.4489	-6.90	**
7-8	0.1607	2.60	**	-0.4293	-4.02	**	-0.3980	-3.98	**
9-10	0.1383	3.36	**	-0.4702	-5.61	**	-0.4408	-5.58	**
11-12	0.0922	5.17	**	-0.5088	-6.21	**	-0.4770	-6.55	**
13-14	0.0958	2.86	**	-0.5154	-8.67	**	-0.4822	-9.60	**
15-16	0.1253	5.41	**	-0.4757	-5.79	**	-0.4447	-5.99	**
17-18	0.1695	10.00	**	-0.4521	-5.71	**	-0.4205	-5.94	**
19-20	0.0763	2.15	*	-0.5367	-5.76	**	-0.5037	-5.94	**
21-22	0.1596	3.71	**	-0.4785	-4.46	**	-0.4452	-4.45	**
23-24	0.1943	3.65	**	-0.4056	-6.77	**	-0.3736	-7.34	**
25-26	0.1670	8.80	**	-0.4728	-6.66	**	-0.4394	-7.19	**
27-28	0.1344	4.25	**	-0.5132	-9.87	**	-0.4794	-11.20	**
29-30	0.1155	5.45	**	-0.5260	-8.43	**	-0.4937	-8.78	**
31-35	0.1154	9.64	**	-0.5274	-6.45	**	-0.4940	-6.68	**
36-40	0.1817	4.74	**	-0.4641	-6.52	**	-0.4298	-6.59	**
41-45	0.1580	6.71	**	-0.5204	-6.92	**	-0.4912	-6.84	**
46-50	0.1062	2.15	*	-0.5654	-5.03	**	-0.5328	-5.26	**
51-60	0.0806	3.54	**	-0.5669	-6.37	**	-0.5398	-6.51	**
61-100	0.0895	3.34	**	-0.4864	-5.74	**	-0.4560	-6.04	**
100-150	0.1048	0.98		-0.4534	-10.39	**	-0.4264	-10.55	**
151-250	0.1862	2.29	*	-0.4680	-4.31	**	-0.4297	-4.24	**
251-409	-0.0208	-0.32		-0.6279	-8.18	**	-0.5917	-8.75	**
ln(AUM)				0.0351	8.72	**	0.0332	9.37	**
Funds for DD							0.0387	4.31	**
Difference	-0.0044	-0.25		0.0415	2.60	**	0.0411	2.60	**
Adjusted R2	0.00			0.09			0.09		
N	922			855			855		

This panel reports results regressing the Sharpe ratio defined using the correction for quarterly smoothing given in Lo (2008) on a dummy variable indicating the number of underlying funds, as well as the log of assets under management and a measure of the maximum available funds for operational due diligence in $M given as the management fee times the ratio of assets under management to the number of funds under management. The Difference variable refers to a similar regression including only dummies for 2-10 funds, 11-25 funds, and 26-409 funds, and the difference and associated *t*-value refers to the difference between the 11-25 fund coefficient and the 26-409 fund coefficient. The reported *t*-values refer to clustered standard errors where observations are clustered according to the stated strategy objective of the fund of funds. * and ** denote significance at the 5% and 1% levels, respectively.

Table 2: Regressing performance measures on degrees of diversification and on assets under management accounting for quarterly smoothing

Panel 2 **Alpha**

Number of Underlying Funds	*Model 1*			*Model2*			*Model 3*		
	Coefficient	*t-value*		*Coefficient*	*t-value*		*Coefficient*	*t-value*	
2	-0.0012	-2.04	*	-0.0083	-6.61	**	-0.0084	-7.47	**
3-4	0.0041	0.64		-0.0101	-5.11	**	-0.0094	-4.50	**
5-6	-0.0012	-1.42		-0.0084	-5.28	**	-0.0077	-4.66	**
7-8	0.0010	0.70		-0.0060	-3.06	**	-0.0053	-2.49	*
9-10	0.0001	0.20		-0.0071	-5.38	**	-0.0064	-4.40	**
11-12	0.0012	1.32		-0.0059	-4.98	**	-0.0052	-4.05	**
13-14	-0.0008	-1.38		-0.0081	-5.03	**	-0.0073	-4.16	**
15-16	-0.0003	-0.35		-0.0075	-5.05	**	-0.0068	-4.34	**
17-18	0.0010	3.61	**	-0.0063	-4.99	**	-0.0056	-3.91	**
19-20	-0.0009	-1.44		-0.0080	-5.35	**	-0.0073	-4.39	**
21-22	0.0005	2.09	*	-0.0070	-6.19	**	-0.0062	-4.94	**
23-24	-0.0004	-0.39		-0.0075	-5.31	**	-0.0068	-4.50	**
25-26	0.0014	3.52	**	-0.0062	-4.89	**	-0.0054	-3.85	**
27-28	-0.0007	-1.69		-0.0085	-4.86	**	-0.0077	-4.07	**
29-30	-0.0007	-1.70		-0.0084	-6.21	**	-0.0076	-5.13	**
31-35	0.0003	0.46		-0.0071	-4.76	**	-0.0063	-3.87	**
36-40	0.0004	0.99		-0.0072	-5.84	**	-0.0064	-4.68	**
41-45	0.0008	12.03	**	-0.0073	-5.16	**	-0.0066	-4.15	**
46-50	-0.0027	-2.97	**	-0.0108	-9.92	**	-0.0100	-8.55	**
51-60	-0.0025	-8.81	**	-0.0099	-7.64	**	-0.0093	-6.48	**
61-100	-0.0015	-1.69		-0.0087	-6.79	**	-0.0080	-6.02	**
100-150	0.0001	0.06		-0.0059	-3.48	**	-0.0053	-3.04	**
151-250	0.0005	0.19		-0.0070	-2.29	*	-0.0061	-1.88	
251-409	-0.0026	-5.73	**	-0.0110	-7.98	**	-0.0101	-6.63	**
ln(AUM)				0.0004	5.82	**	0.0004	4.66	**
Funds for DD							0.0009	3.99	**
Difference	0.0005	2.34	*	0.0011	4.18	**	0.0011	4.13	**
Adjusted R2	0.02			0.05			0.06		
N	922			855			855		

This panel reports results regressing the Fung and Hsieh alpha for the fund (corrected for quarterly smoothing by including factors and two lags of factors allowing for an MA(2) process for the errors, as described in Lo (2008)) on a dummy variable indicating the number of underlying funds, as well as the log of assets under management and a measure of the maximum available funds for operational due diligence in $M given as the management fee times the ratio of assets under management to the number of funds under management. The Difference variable refers to a similar regression including only dummies for 2-10 funds, 11-25 funds and 26-409 funds, and the difference and associated t-value refers to the difference between the 11-25 fund coefficient and the 26-409 fund coefficient. The reported t-values refer to clustered standard errors where observations are clustered according to the stated strategy objective of the fund of funds.

Table 2: Regressing performance measures on degrees of diversification and on assets under management accounting for quarterly smoothing

Panel 3 **Appraisal_Ratio**

Number of underlying funds	*Model 1* *Coefficient*	*t-value*		*Model2* *Coefficient*	*t-value*		*Model 4* *Coefficient*	*t-value*	
2	-0.0087	-2.44	*	-0.0600	-8.05	**	-0.0608	-9.36	**
3-4	0.0133	0.43		-0.0711	-5.72	**	-0.0647	-4.85	**
5-6	-0.0066	-1.54		-0.0583	-6.36	**	-0.0526	-5.38	**
7-8	0.0052	0.73		-0.0455	-4.15	**	-0.0390	-3.23	**
9-10	0.0008	0.27		-0.0508	-6.43	**	-0.0446	-5.07	**
11-12	0.0071	1.20		-0.0448	-6.88	**	-0.0381	-5.46	**
13-14	-0.0036	-0.83		-0.0558	-5.14	**	-0.0488	-4.07	**
15-16	0.0003	0.06		-0.0512	-5.45	**	-0.0447	-4.45	**
17-18	0.0088	3.62	**	-0.0443	-5.43	**	-0.0377	-4.03	**
19-20	-0.0053	-1.23		-0.0568	-5.72	**	-0.0499	-4.52	**
21-22	0.0044	2.31	*	-0.0495	-7.35	**	-0.0426	-5.61	**
23-24	0.0007	0.10		-0.0510	-5.78	**	-0.0443	-4.72	**
25-26	0.0085	3.74	**	-0.0456	-5.85	**	-0.0386	-4.41	**
27-28	-0.0055	-2.41	*	-0.0613	-5.99	**	-0.0543	-4.80	**
29-30	-0.0024	-0.88		-0.0576	-7.00	**	-0.0508	-5.48	**
31-35	0.0017	0.46		-0.0515	-5.20	**	-0.0445	-4.06	**
36-40	0.0032	1.16		-0.0511	-7.00	**	-0.0439	-5.34	**
41-45	0.0071	13.30	**	-0.0513	-5.67	**	-0.0452	-4.28	**
46-50	-0.0162	-2.49	*	-0.0739	-12.25	**	-0.0671	-10.47	**
51-60	-0.0151	-10.97	**	-0.0701	-9.00	**	-0.0645	-7.24	**
61-100	-0.0095	-1.71		-0.0614	-7.46	**	-0.0550	-6.37	**
100-150	0.0014	0.13		-0.0424	-4.06	**	-0.0368	-3.49	**
151-250	0.0052	0.28		-0.0492	-2.22	*	-0.0412	-1.74	
251-409	-0.0135	-12.62	**	-0.0722	-8.38	**	-0.0647	-6.64	**
ln(AUM)				0.0030	6.69	**	0.0026	5.13	**
Funds for DD							0.0081	4.39	**
Difference	0.0041	3.07	**	0.0085	5.53	**	0.0084	5.47	**
Adjusted R2	0.02			0.06			0.07		
N	922			855			855		

This panel reports results regressing the appraisal ratio defined using the quarterly smoothing-adjusted alpha described in Panel 2 on a dummy variable indicating the number of underlying funds, as well as the log of assets under management and a measure of the maximum available funds for operational due diligence in $M given as the management fee times the ratio of assets under management to the number of funds under management. The Difference variable refers to a similar regression including only dummies for 2-10 funds, 11-25 funds and 26-409 funds, and the difference and associated t-value refers to the difference between the 11-25 fund coefficient and the 26-409 fund coefficient. The reported t-values refer to clustered standard errors where observations are clustered according to the stated strategy objective of the fund of funds.

Table 3: Fraction of Funds that survive the Due Diligence Test

Number of Underlying Funds	***N***	***Average***	***t-value***	
2	44	32%	-9.71	**
3 – 4	18	6%	-17.49	**
5 – 6	102	17%	-22.58	**
7 – 8	42	24%	-11.59	**
9 – 10	71	30%	-13.00	**
11 – 12	68	10%	-24.34	**
13 – 14	52	13%	-18.28	**
15 – 16	95	13%	-25.63	**
17 – 18	65	18%	-16.94	**
19 – 20	96	15%	-23.71	**
21 – 25	161	19%	-26.51	**
26 – 30	158	22%	-23.56	**
31 – 40	124	20%	-22.16	**
41 – 50	74	24%	-15.17	**
51 – 409	75	23%	-16.00	**

This table presents a due diligence test by inspecting whether for a FoHFs $\frac{\$US\ Assets\ under\ management}{Number\ of\ underlying\ hedge\ funds} \times Management\ fee > \$12{,}500$. Under the null hypothesis, all funds in the sample pass the test. The large t-values strongly reject the null hypothesis in all instances suggesting that a significant fraction of all funds fail this test. * and ** denote significance at the 5% and 1% levels, respectively.

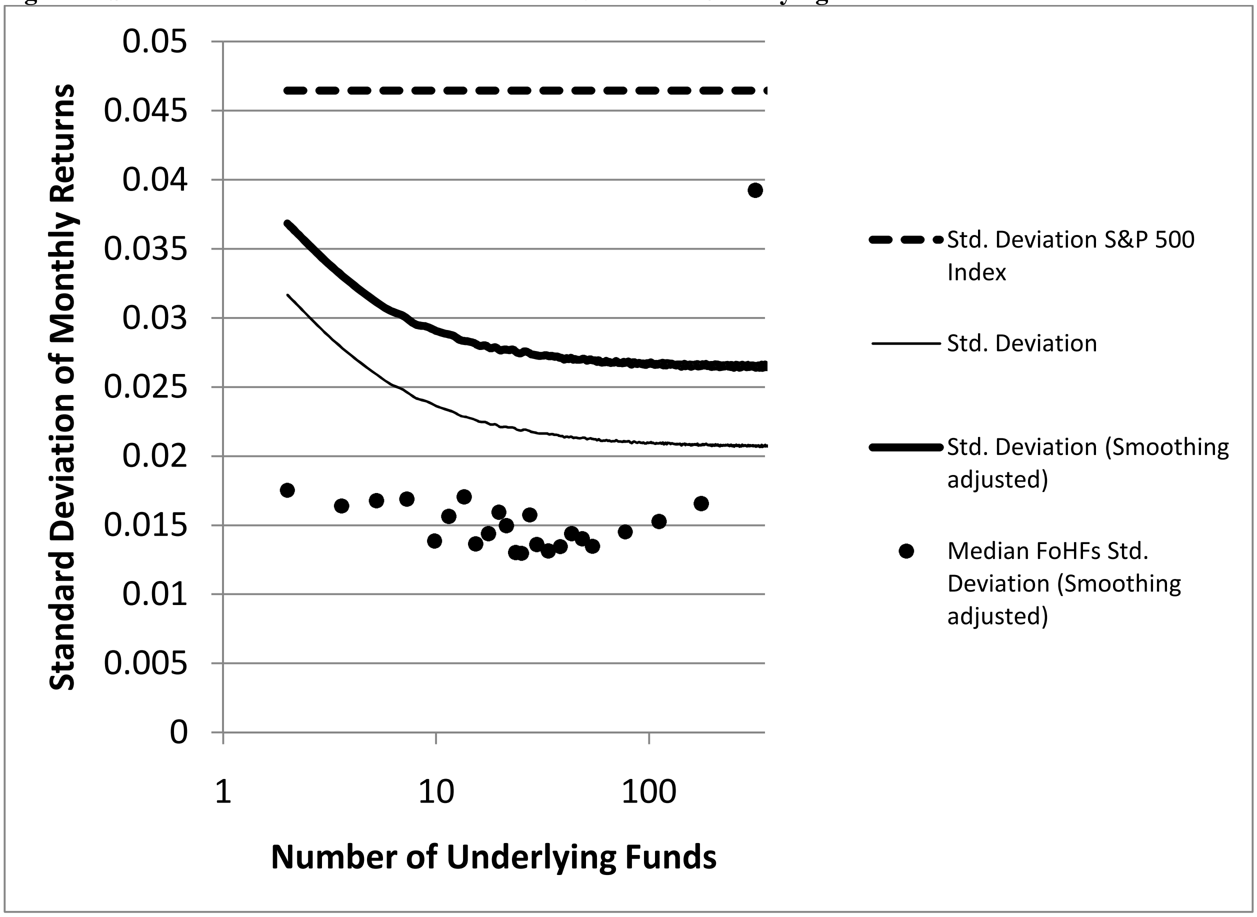
Table 4: Cox Proportional Hazard analysis of the death of FoHFs

	Model 1			*Model 2*			*Model 3*		
	Coefficient	*t-value*		*Coefficient*	*t-value*		*Coefficient*	*t-value*	
Log(AUM)	-0.0301	-2.52	*	0.0479	3.17	**	0.0422	2.79	**
Age				-0.0001	-9.28	**	-0.0001	-9.34	**
Number of Funds				-0.1609	-6.63	**	-0.1677	-6.88	**
Log(std.dev)				0.1272	4.23	**	0.1178	3.90	**
Funds for DD ($M)				-0.1251	-2.70	**	-0.1184	-2.58	*
Skewness							-0.2585	-7.87	**
Log Likelihood	-23169.74			-23030.25			-22999.26		

This table presents a standard Cox Proportional Hazards model explaining the hazard rate of FoHFs for each year of our sample based on data measured prior to that year. A positive coefficient implies that the associated explanatory variable increases the hazard rate above the mean or baseline hazard rate for the sample. At the beginning of each year we gather the characteristics of the fund as of the end of the prior year and examine the remaining life of the fund allowing for right truncation (i.e., we do not know the remaining life of funds that survive to the end of our dataset). Age is measured in the number of days since the inception of the fund as of the record date. Funds for DD in $M is given as the management fee times the ratio of assets under management in $M (as of the immediately prior December record date) to the number of funds under management and is to be interpreted as the maximum available funds for this purpose. Skewness and standard deviation are measured with the appropriate adjustment for quarterly smoothing. We consider a fund dead if it has been liquidated, delisted, or has an unknown status and it has stopped reporting to the database. * and ** denote significance at the 5% and 1% levels, respectively.

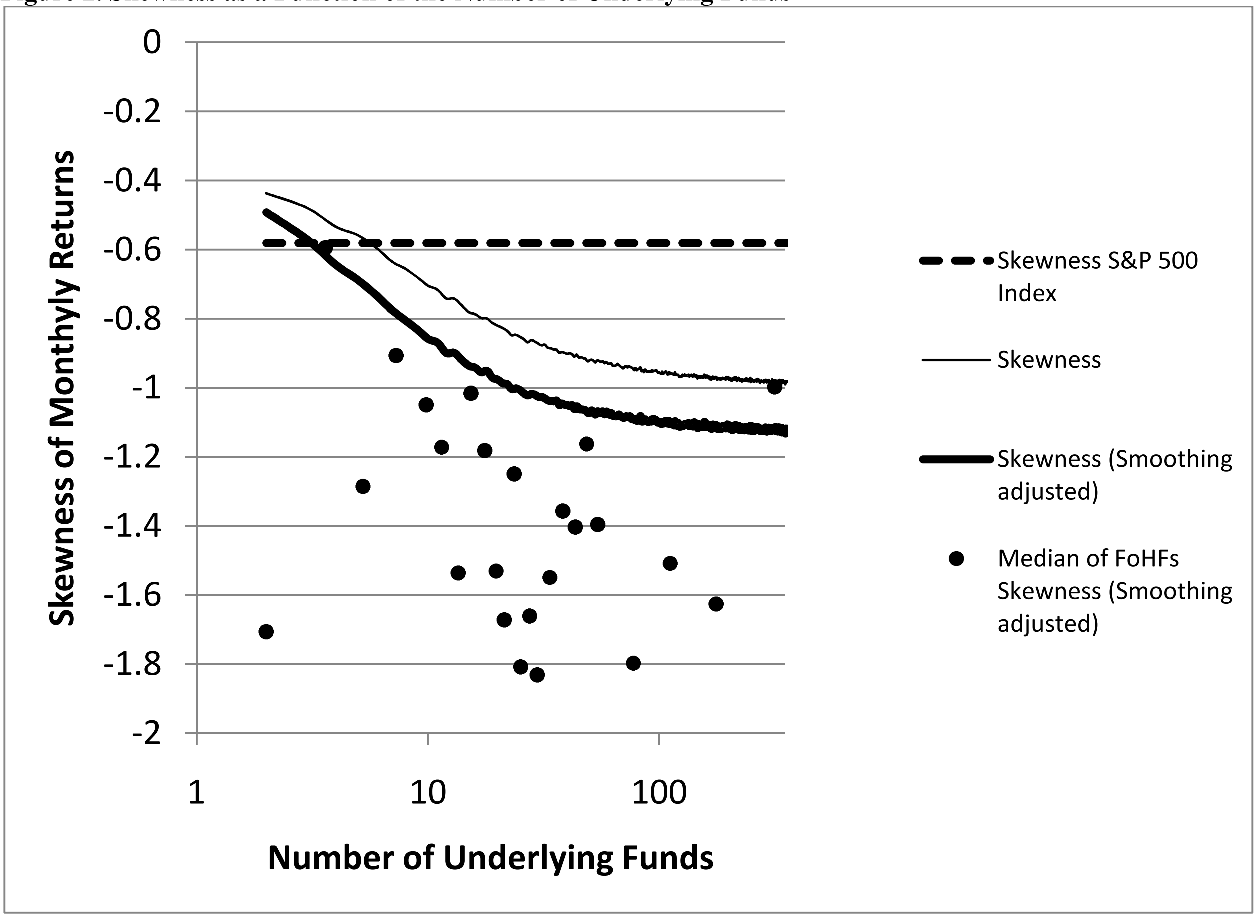
Figures

Figure 1. Standard Deviation as a Function of the Number of Underlying Funds



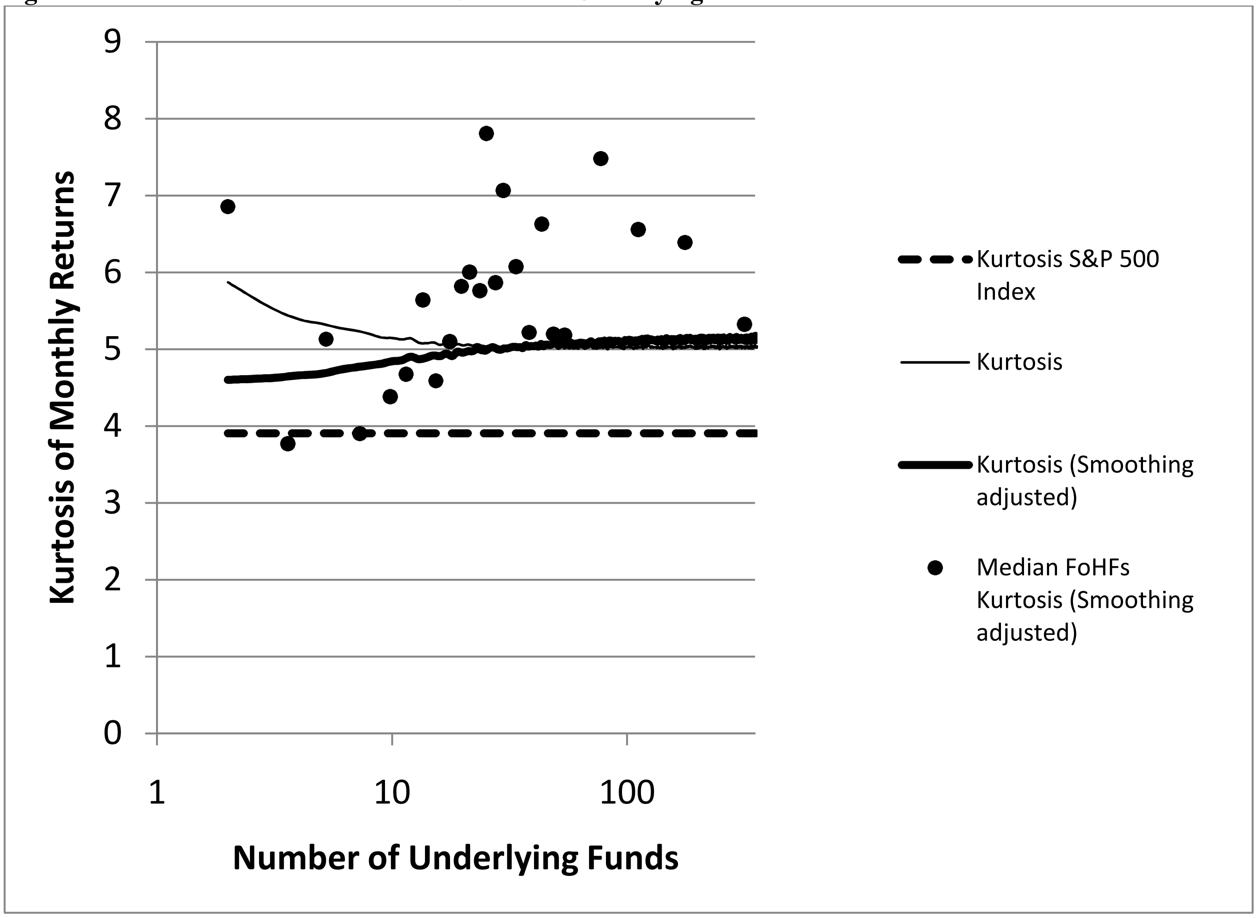
This figure reports the average standard deviation of holding period returns net of fees of 25,000 randomly chosen portfolios ranging from two to 350 underlying hedge funds over 60 months in the interval from January 2000 through March 2010. We also report the average standard deviation appropriately adjusted for smoothing using the procedures outlined in Getmansky, Lo, and Makarov (2004). Superimposed on this figure is the median standard deviation for FoHFs within ranges of the number of underlying hedge funds. For reference we also give the standard deviation of the total return on the S&P 500 index from January 2000 through March 2010.

Figure 2. Skewness as a Function of the Number of Underlying Funds



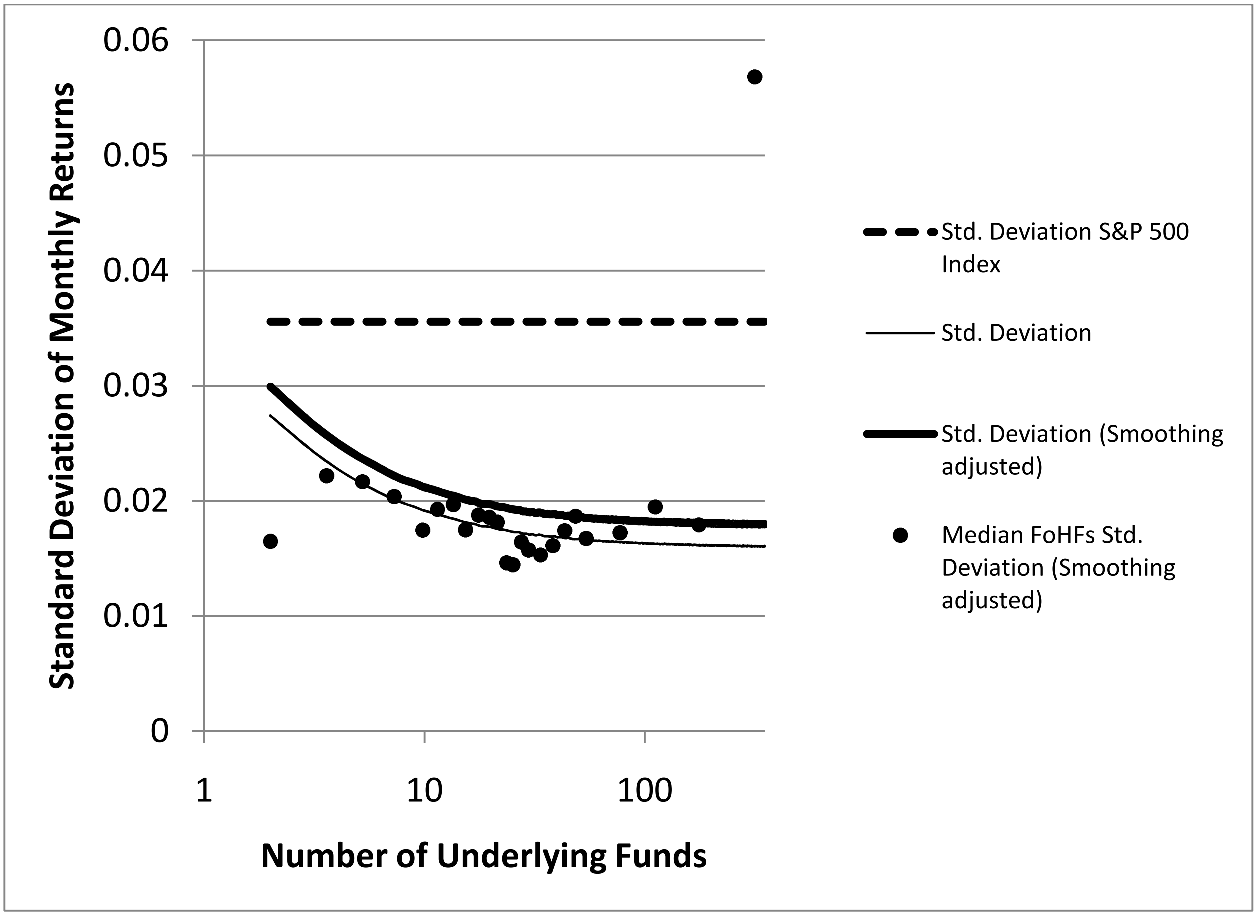
This figure reports the average skewness of holding period returns net of fees of 25,000 randomly chosen portfolios ranging from two to 350 underlying hedge funds over 60 months in the interval from January 2000 through March 2010. We also report the average skewness appropriately adjusted for smoothing using extensions of the procedures outlined in Getmansky, Lo, and Makarov (2004). Superimposed on this figure we give the median skewness for FoHFs in our sample within ranges of the number of underlying hedge funds. For reference we also give the skewness of the total return on the S&P 500 index from January 2000 through March 2010.

Figure 3. Kurtosis as a Function of the Number of Underlying Funds



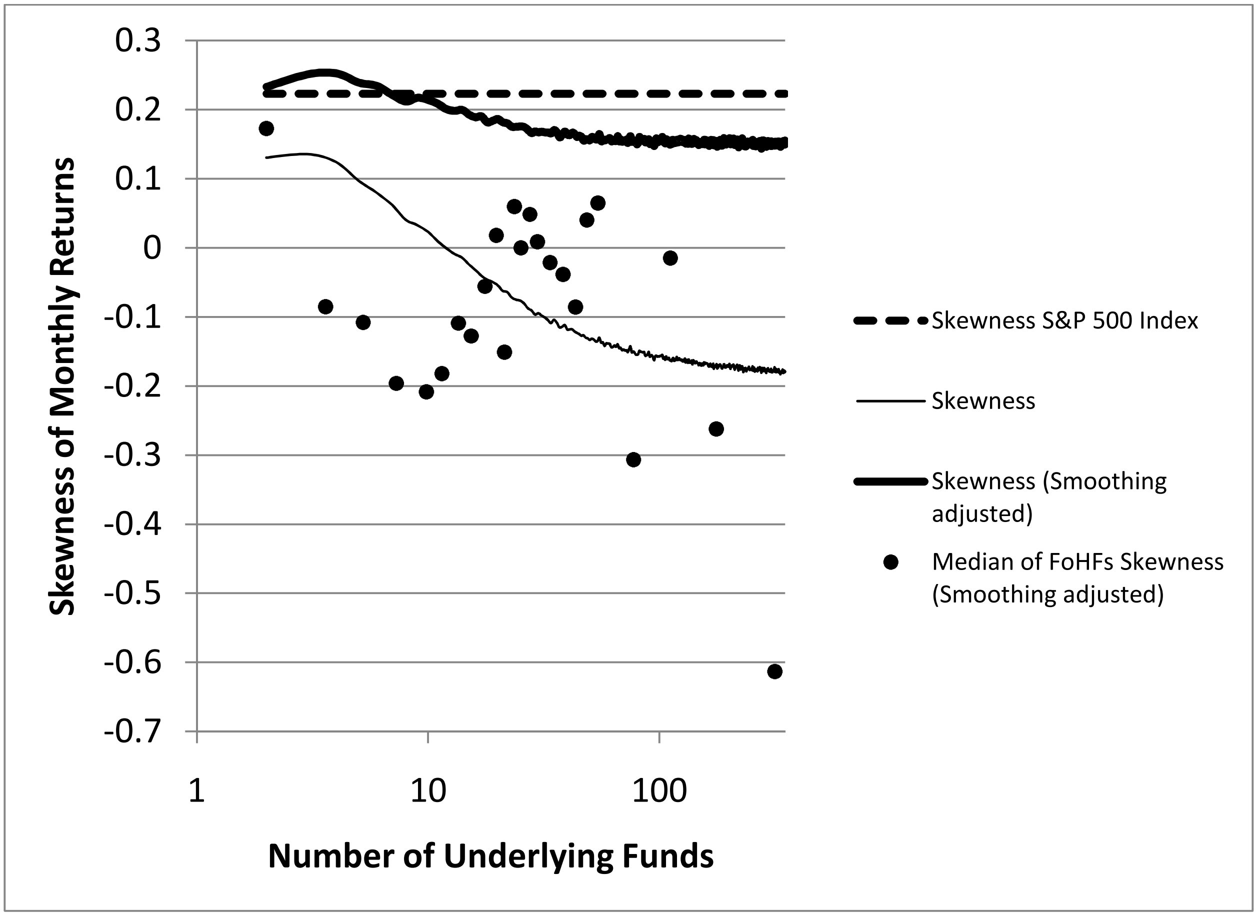
This figure reports the average kurtosis of holding period returns net of fees of 25,000 randomly chosen portfolios ranging from two to 350 underlying hedge funds over 60 months in the interval from January 2000 through to March 2010. We also report the average kurtosis appropriately adjusted for smoothing using extensions of the procedures outlined in Getmansky, Lo, and Makarov (2004). Superimposed on this figure we give the median kurtosis for FoHFs within ranges of the number of underlying hedge funds. For reference we also give the kurtosis of the total return on the S&P 500 index from January 2000 through March 2010.

Figure 4. Standard Deviation as a Function of the Number of Underlying Funds Excluding Market Left Tail Events



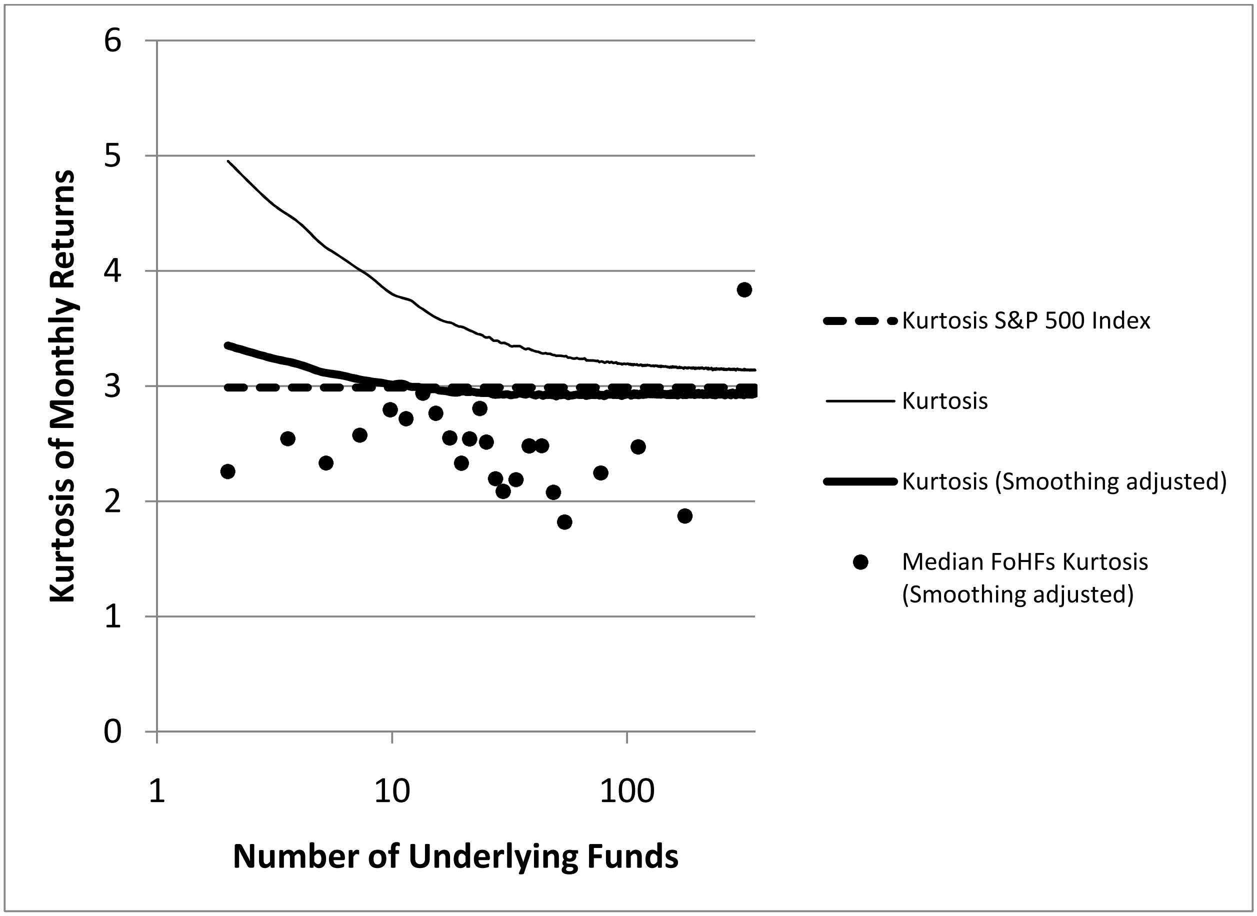
This figure reports the average standard deviation of holding period returns net of fees of 25,000 randomly chosen portfolios ranging from two to 350 underlying hedge funds over 60 months in the interval from January 2000 through March 2010 excluding market left tail events defined as months in which the S&P 500 return in excess of the one-month T-bill return fell below its tenth decile over this sample period. We also report the average standard deviation appropriately adjusted for smoothing using the procedures outlined in Getmansky, Lo, and Makarov (2004) excluding these months. Superimposed on this figure we give the median standard deviation for FoHFs in our sample within ranges of the number of underlying hedge funds again excluding these months. For reference we also give the standard deviation of the total return on the S&P 500 index from January 2000 through March 2010 excluding these market left tail events.

Figure 5. Skewness as a Function of the Number of Underlying Funds Excluding Market Left Tail Events



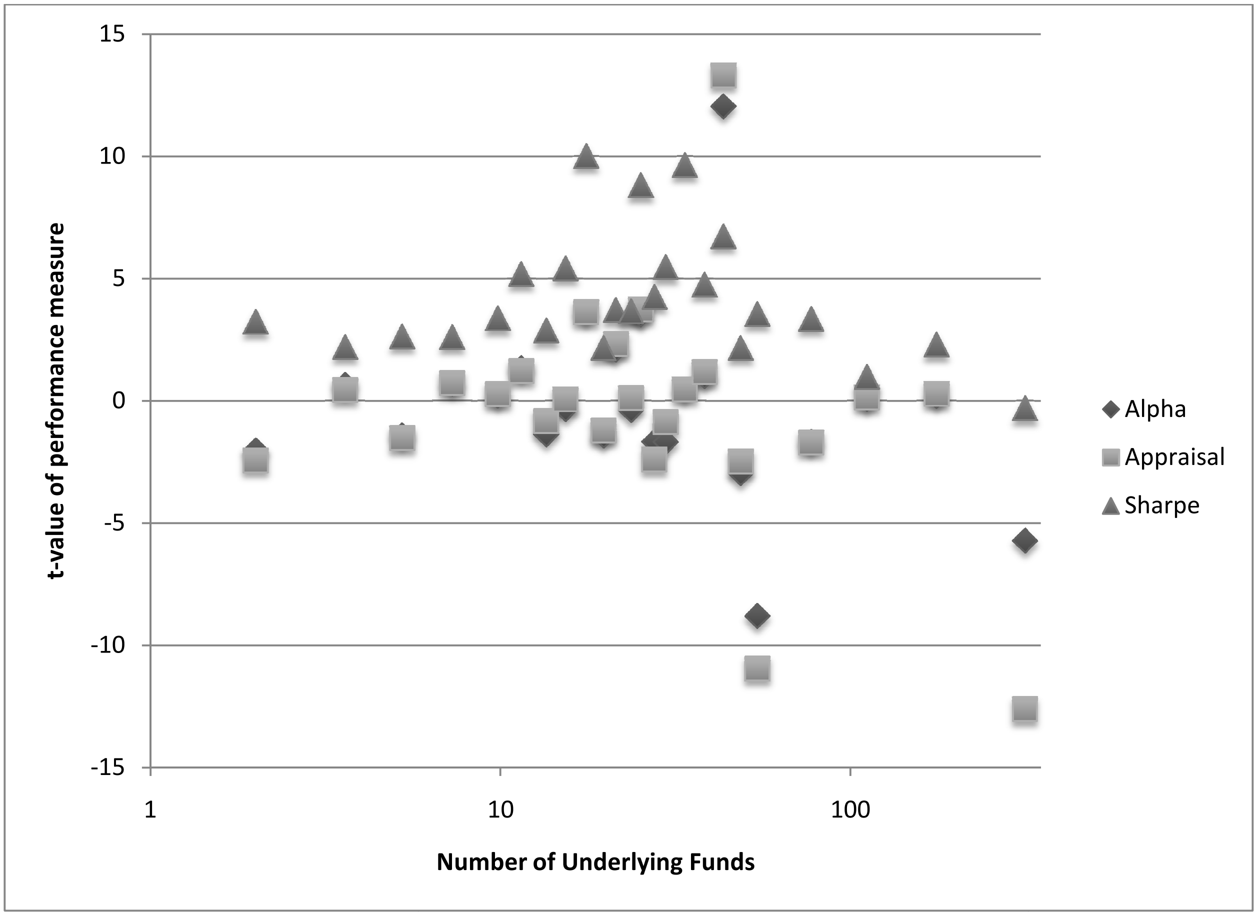
This figure reports the average skewness of holding period returns net of fees of 25,000 randomly chosen portfolios ranging from two to 350 underlying hedge funds over 60 months in the interval from January 2000 through March 2010 excluding market left tail events defined as months in which the S&P 500 return in excess of the one-month T-bill return fell below its tenth decile over this sample period. We also report the average skewness appropriately adjusted for smoothing using extensions of the procedures outlined in Getmansky, Lo, and Makarov (2004) excluding these months. Superimposed on this figure we give the median skewness for FoHFs within ranges of the number of underlying hedge funds again excluding these months. For reference we also give the skewness of the total return on the S&P 500 index from January 2000 through March 2010 excluding these market left tail events.

Figure 6. Kurtosis as a Function of the Number of Underlying Funds Excluding Market Left Tail Events



This figure reports the average kurtosis of holding period returns net of fees of 25,000 randomly chosen portfolios ranging from two to 350 underlying hedge funds over 60 months in the interval from January 2000 through March 2010 excluding market left tail events defined as months in which the S&P500 return in excess of the one-month T-bill return fell below its tenth decile over this sample period. We also report the average kurtosis appropriately adjusted for smoothing using extensions of the procedures outlined in Getmansky, Lo, and Makarov (2004) excluding these months. Superimposed on this figure we give the median kurtosis for FOFs in our sample within ranges of the number of underlying hedge funds again excluding these months. For reference we also give the kurtosis of the total return on the S&P 500 index from January 2000 through March 2010 excluding these market left tail events.

Figure 7: Performance as a Function of the Number of Underlying Funds



This figure presents the *t*-values of the three different measures of performance given in Panels 1, 2, and 3 of Table 2 as a function of the number of underlying hedge funds. These measures of performance are given as the coefficients of a regression of the measures of performance on dummy variables indicating the number of underlying hedge funds in each FoHF. The *t*-values are computed on the basis of standard errors clustered by stated strategic objective of the FoHFs.

Footnotes

[1] Over the ten-year (1998-2008) period, foundations and endowment funds increased their exposure to hedge funds from 2.8% to 12.9% (http://www.northerntrust.com/pointofview/09_Fall_Winter/megatrends.html). We expect that most of this institutional growth is through FoHFs which grew exponentially both in number and assets under management over that period.

[2] For example, New York State Estates, Powers and Trusts Law section 11-2.3c.

[3] If an operational due diligence on any one underlying hedge fund costs the FoHF $12,500 (Brown et al. 2012), then with a 1.5% management fee, a FoHFs with 100 funds would need at least $83.3 million assets under management to perform necessary due diligence. Note this does not account for any other management expenses. The $12,500 number is a minimum fee assuming that the client is willing to share the results of the due diligence with other clients.

[4] We use the December 2010 Barclay Hedge reports, but limit our attention to U.S. dollar funds prior to the second quarter of 2010 due to the fact that many funds delay reporting to the database. The database reports AUM in U.S. dollars for all funds, but not necessarily for performance. However, many, but by no means all, non-U.S. dollar funds report returns in U.S. dollars. Some funds report performance on multiple currency bases and for this reason we limit our attention to U.S. dollar funds.

[5] See, for example, Malkiel and Saha (2005) who indicate that this is a particular problem for hedge fund database reported returns prior to 2001.

[6] Out of 3,767 FOFs, we have 2,144 U.S. dollar funds and 947 Euro funds.

[7] We exclude from the sample all data within 18 months of the inception date of the fund to address concerns about incubation bias. Many funds start reporting data considerably after the

inception date. In later results we exclude 44 of these funds that report just one underlying hedge fund, and in addition require at least 36 months of data to compute the appropriate smoothing adjustments for different performance measures we report.

[8] This is a standard formula, given for example in Lo's (2008, p. 80) Equation 3.46. The standard deviation of monthly returns is given by the square root of a third of the quarterly variance computed using this formula.

[9] The relatively high standard deviation of the most diversified group of FoHFs with from 264 to 409 underlying funds may be an artifact of each of them having died within the sample with all but one having sequential monthly returns less than -10% in the final quarter of their life.

[10] An interesting question is the effect of diversification on residual risk as measured by the residuals estimated using the Fung and Hsieh (2004) factors described in the next section. The interesting result is that there is no result: diversification has no effect on residual risk measures. We attribute this finding to the fact that the FoHFs in our sample are not random selections of hedge funds, but hedge funds chosen with past high returns and possibly correlated residual risk components. This hypothesis is consistent with the result that residuals are highly cluster correlated by strategy definition of the FoHFs.

[11] Amin and Kat (2002) use a similar approach based on data from 1994 through 2001 and using 500 simulations of portfolios of from 2 to 20 randomly chosen funds. They find some evidence that skewness decreases with the number of randomly chosen hedge funds approaching a level consistent with the skewness of the S&P 500 return, while kurtosis is relatively unaffected by diversification and is less than that of the contemporary S&P 500 return.

[12] We assume stationarity and that all relevant time covariates depend only on time displacement to compute smoothing-adjusted skewness and kurtosis. Then by a simple (if tedious) extension in

Lo's (2008, p. 80) Equation 3.46, we can define the third central moment which when divided by the cube of the smoothing adjusted standard deviation and the fourth central moment that divided by the fourth power of the smoothing adjusted standard deviation gives rise to the smoothing adjusted skewness and kurtosis measures given in Figures 2 and 3. We treat the market left-tail event months as missing data in computing necessary time covariates to compute the smoothing adjusted skewness and kurtosis reported in Figures 5 and 6.

[13] Over the same time period, the skewness of the Pástor and Stambaugh (2003) liquidity innovation was -.73, based on updated measures for the period of our data, January 2000 through March 2010 obtained through WRDS. Using data available only through December 2008 through WRDS, the Sadka (2006) permanent variable liquidity factor found most related to asset pricing had a negative skew of -1.47. While both the Amihud (2002) and the Amihud liquidity innovations factor defined in Acharya and Pedersen (2005) have positive skewness (using data kindly provided by Yakov Amihud), the Pastor/Stambaugh and Sadka measures are well within the 95% confidence limits defined in terms of a bootstrap of the skewness measure based on a million samples of 60 observations taken with replacement from the Amihud illiquidity measure and the Amihud innovations.

[14] This result is, in this limited hedge fund context, a counterexample to the general presumption that diversification reduces left-tail risk exposure. If anything, diversification exacerbates this measure of risk. If left-tail risk is not diversifiable in a more general context, Bawa and Lindenberg (1977) and Harvey and Siddique (2000) observe this may have significant asset pricing implications.

[15] This point is made in Lin (2011). Goetzmann et al. (2007) make the observation that strategies that increase short-term performance at the expense of increased tail risk exposure will in general

generate Sharpe ratios in excess of the benchmark. In this context this will create an adverse incentive for both hedge fund and FoHFs managers who fail to properly account for tail risk exposure of their funds.

[16] We also considered a measure of left-tail risk due to Bawa and Lindenberg (1977) employed by Chen (2011) in this context. While by this measure there is a reduction in left-tail risk moving from two underlying funds to a greater degree of diversification, left-tail risk increases with the degree of diversification beyond 24 underlying hedge funds, consistent with the results reported in Figures 2 and 3.

[17] In correcting for smoothing, it is important to consider the excluded months as "missing data" in constructing the relevant time covariates.

[18] We thank David Hsieh for making this data available at his website. See http://faculty.fuqua.duke.edu/~dah7/HFData.htm.

[19] Most of the literature that employs the Fung/Hsieh factors interprets the *Bond Factor* and the *Credit Spread* as the change in the monthly market yield of the 10-year Treasury constant maturity yield and the monthly change in the Moody's Baa yield less the 10-year Treasury constant maturity yield, respectively. See, for example Kosowski, Naik, and Teo (2007) and the references cited there. However, these are not traded factors and so we define the *Bond Factor* as the total return for 10-year government bonds reported in CRSP in excess of the one-month T-bill return and the *Credit Spread* as the difference in total return between the total return on the Bank of America Merrill Lynch US Corporate Bond Index and the total return on the 10-year government bonds reported in CRSP. This change does not in any way affect any of the results we report in this paper.

[20] Brown et al. (1992) argue that in addition the use of the appraisal ratio mitigates the look-

ahead bias caused by the requirement that the fund survive at least 36 months to be included in our sample of performance.

[21] On an annual basis, this is given by:

$$Management\ fee\ \times$$
$$U.S.\,Dollar\ Assets\ under\ management \big/ number\ of\ underlying\ funds.$$

[22] Many FoHFs perform their own internal due diligence and do not outsource this to hedge fund consultants. Informal discussions with FoHFs that perform their own internal due diligence suggests that properly allocating management costs to the due diligence function, \$50,000 may be a significant underestimate of the true cost of the due diligence function.